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Control of a team of car - like robots using abstractions

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Abstract

We present a method to coordinate a large number of under - actuated robots by designing control laws on a small dimensional manifold, independent on the number and ordering of the robots. The small dimensional description of the team has a product structure of a Lie group, which captures the dependence of the ensemble on the world frame, and a shape manifold, which is an intrinsic description of the team. We design decoupled controls for group and shape. The individual control laws which are mapped to the desired collective behavior can be realized by feedback depending only on the current state of the robot and the state on the small dimensional manifold, so that the robots have to broadcast their states and only have to listen to some coordinating agent with small bandwidth.

1 Introduction

We approach the problem of controlling a large number of car -like robots required to accomplish a mission as a team. For example, consider the task of moving hundreds of robots from arbitrary initial positions through a tunnel while staying grouped so that the distance between each pair does not exceed a certain value. The simplest solution, generating motion plans or control laws for each robot, is obviously not feasible from a computational viewpoint. It is desired to have a certain level of abstraction: the motion generation/control problem should be solved in a lower dimensional space which captures the behavior of the group and the nature of the task.

The robots can be required to form a *virtual structure*. In this case, the problem is reduced to a left invariant control system on $SE(l)$ ($l = 1, 2$), and the individual trajectories are $SE(l)$ - orbits [1]. The literature on stabilization and control of virtual structures is rather extensive. Most of the recent works model formations using *formation graphs*, which are graphs whose nodes capture the individual agent kinematics or dynamics, and whose edges represent inter-agent constraints that must be satisfied [9]. Characterizations of rigid formations can be found in [6, 1]. The controllers guaranteeing local asymptotic stability of a given rigid formation are derived using Lyapunov energy-type functions [9]. Examples of such functions include positive definite convex *formation functions* [5] and biologically inspired *artificial potential functions* [8]. The global minima of such functions exhibit $SE(l)$, $l = 1, 2, 3$ symmetry and also expansion/contraction symmetries, which can be used to decouple the mission control problem into a formation keeping subproblem and a maneuver subproblem [8].

The virtual structure approach is not appropriate for many applications, including obstacle avoidance, tunnel passing, etc. Also, the rigid formulation is based on identified robots, which makes the obtained control laws and motion plans invalid in the case of individual failures. Moreover, the rigidity constraint induces an inherent coupling between the control systems on the symmetry group and the shape space. For example, in [8], the authors have to limit the speed of convergence on the symmetry group so that, while moving as a group, the individual agents do not leave the local regions of attractions guaranteeing convergence to the desired shape.

In this paper we build on [2]. The robots are modeled as underactuated kinematic drift - free control systems. We define outputs which give the cartesian coordinates of some reference points on the robots, which are used in the formulation of the collective tasks, i.e.,, in a cooperative mission, the robots are represented by their reference points. Using input - output feedback linearization for each robot, the controls are related to the velocities of the output through a linear nonsingular map. We then propose an abstraction based on the definition of a map ϕ from the space of all robot outputs to a lower dimensional *abstraction manifold* A. We require the abstraction manifold to have a product structure $A = G \times S$, where G is a *Lie group* that captures the dependence of the problem on the chosen world coordinate frame and S is a *shape manifold*, which is an intrinsic description of the team. We also impose that the map ϕ is so that each abstract variable can be controlled independently, so that the user can easily design controllers to only change the shape for example, and keep the group variable fixed. In this paper, G is $SE(2)$, the special Euclidean group in two dimensions, and S gives a description of the distribution of the robots along the axes of a virtual frame whose pose on the world frame evolves on G. The task to be accomplished by the team suggests a natural feedback control system on A. We show that the individual control laws

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which are mapped to the desired team behavior can be realized by feedback depending only on the current state of the robot and the state on the abstraction manifold, so that the robots have to broadcast the coordinates of their reference points and only have to listen to some coordinating agent with small bandwidth.

2 Problem formulation

Consider N identical car - like planar robots. In the world frame $\{W\}$, robot i is described by a 3 - dimensional state vector $x^i = [x_1^i, x_2^i, x_3^i]^T$, $i = 1, \ldots N$, where (x_1^i, x_2^i) give the cartesian coordinates of the robot center and x_3^i measures the orientation of the robot frame in $\{W\}$. Each robot is modeled as a kinematic, drift free control system

$$\dot{x}^i = G(x^i)u^i = g_1(x^i)u_1^i + g_2(x^i)u_2^i \tag{1}$$

where the control vector fields are given by

$$g_1(x) = \begin{bmatrix} \cos x_3 \\ \sin x_3 \\ 0 \end{bmatrix} \quad g_2(x) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{2}$$

for $x = [x_1, x_2, x_3]^T$. The control $u^i = (u_1^i, u_2^i)$ consists of driving and steering speeds. On each robot we pick a reference point P_i different from the robot center and with coordinates $(d, 0)$ in the robot frame. The cartesian coordinates $q^i = (q_1^i, q_2^i)$ of the reference points are used to formulate cooperative tasks. In other words, for each robot i, $i = 1, \ldots, N$, we define an output map

$$q^i = h(x^i) \tag{3}$$

where h is given by

$$h(x) = \begin{bmatrix} x_1 + d\cos x_3 \\ x_2 + d\sin x_3 \end{bmatrix} \tag{4}$$

Problem 1 (Control) *Design control laws u^i, $i = 1, \ldots, N$ so that the team of N robots accomplishes a cooperative task formulated in terms of the chosen reference points q^i.*

Examples of tasks include stabilization inside a given region of the space, tunnel passing, expansions and contractions, etc. Before we start, note that the choice of output function h together with the linearity of dynamics (1) in u^i leads to a linear nonsingular relationship between the derivative of the output $\dot{q}^i$ and the control variables $\dot{q}^i = dh(x^i)G(x^i)u^i$, unless $d = 0$. dh denotes the differential of h. Therefore, we can define new inputs

$$\dot{q}^i = v^i \tag{5}$$

which are related to the original ones by

$$u^i = A(x^i)v^i \tag{6}$$

where

$$A(x) = (dh(x)G(x))^{-1} = \begin{bmatrix} \cos x_3 & \sin x_3 \\ -\frac{\sin x_3}{d} & \frac{\cos x_3}{d} \end{bmatrix} \tag{7}$$

Equations (1), (3), (5) and (6) represent an input output feedback linearization problem [7]. The next natural step would be to set $v^i = \dot{q}^{id} + k(q^{id} - q^i)$, $k > 0$ so that q^i exponentially tracks a given desired trajectory $q^{id}(t)$. Alternatively, in the next section we show how the redefined inputs v^i can be designed so that the robots described by the reference points q^i, $i = 1, \ldots, N$ have a desired collective behavior.

3 Abstraction: general considerations

Collect all the robot outputs q^i and redefined inputs v^i together into a $2N$-dimensional control system

$$\dot{q} = v, \; q \in Q, \; v \in V \tag{8}$$

where

$$\begin{aligned} Q &= \{q | = (q^1, \ldots, q^N) \in \mathbb{R}^{2N}\} \\ V &= \{v | = (v^1, \ldots, v^N) \in \mathbb{R}^{2N}\} \end{aligned} \tag{9}$$

and allow to recover the individual states and controls by using the canonical projection

$$\pi_i(q) = q^i, \; \pi_i(v) = v^i, \; i = 1, \ldots, N \tag{10}$$

Given a large number of robots represented by the cartesian coordinates q_i of the chosen reference points P_i, or, equivalently, a $q \in Q$, we want to solve motion generation / control problems on a smaller dimensional space, which captures the essential features of the group, according to the class of tasks to be accomplished.

The abstraction is based on the definition of a map

$$\phi : Q \to A, \; \phi(q) = a \tag{11}$$

which satisfies the following properties:

(i) The map ϕ is a surjective submersion.

(ii) The map ϕ is invariant to permutations of the robots and the dimension n of A is not dependent on the number of robots N.

(iii) We require that A have a product structure

$$A = G \times S, \; a = (g, s), \; \phi = (\phi_g, \phi_s) \tag{12}$$

where G is a Lie group. An arbitrary $g \in G$ is called *group*, or *pose* and an $s \in S$ is called *shape.*

(iv) The control systems on the group G and shape S are decoupled.

(v) The amount of inter - robot communication in the overall control architecture is limited.

Instead of designing motion on the high dimensional space Q, we want to be able to describe collective behaviors in terms of time - parameterized curves on the small dimensional manifold A. This problem can be treated as an input - output linearization problem for the system described by (8) and (11). Let the new inputs be denoted by $w \in \mathbb{R}^n$. Then

$$\dot{a} = w \tag{13}$$

and, since $w = d\phi(q)v$, we have

$$v = d\phi^T(d\phi d\phi^T)^{-1}w \tag{14}$$

Note that the submersion condition (i) on ϕ implies the surjectivity of the differential $d\phi$ at any $q \in Q$, which guarantees the existence of controls v pushed forward to any small dimensional control w on the abstraction manifold A. Moreover, $\phi_1, \ldots, \phi_n$ are functionally independent, or, equivalently, $d\phi = (d\phi_1, \ldots, d\phi_N)$ is full row rank and the inversion in (14) is well defined ($d\phi_i$ should be interpreted as rows giving the coordinates of the corresponding differential one form and $d\phi$ is the row span of $d\phi_i$'s).

We want the dimension of the control problem to be independent of the number of agents and also independent on possible ordering of the robots. Requirement (ii) will provide control laws which are robust to individual failures and also good scaling properties.

The main idea of requirement (iii) is to have a control suited description of the team of robots a in terms of the pose g of a virtual structure, which captures the dependence of the team on the world frame $\{W\}$, plus a shape s, which is decoupled from g, and therefore, an intrinsic property of the formation. In other words, if $\bar{g}$ is an arbitrary element of G, we require the map ϕ to satisfy

$$\phi(q) = (g, s) \Rightarrow \phi(\bar{g}q) = (\bar{g}g, s) \tag{15}$$

where $\bar{g}q$ represents the action of the group element $\bar{g}$ on the configuration $q \in Q$ and $\bar{g}g$ represents the left translation of g by $\bar{g}$ using the composition rule on the group G. Since we only approach planar robots in this paper, G is $SE(2)$. $\bar{g}q$ represents a rigid displacement of all the robots by $\bar{g}$. (15) is a left invariance - type property of the map ϕ, which gives invariance of our to be designed control laws to the pose of the world frame $\{W\}$. Indeed, if the world frame $\{W\}$ is displaced by $\bar{g}$, the shape s is not affected while the pose g is left translated by $\bar{g}$.

If requirement (iv) is satisfied, then one can design control laws for the interest variables $a \in A$ separately, *e.g*, change the pose of the formation g while preserving the shape s. The control systems on G and S are decoupled if $d\phi_g$ and $d\phi_s$ are orthogonal as subspaces (we assume that Q is equiped with an Euclidean metric). Moreover, if $d\phi_i$'s are all orthogonal, from (14) we have

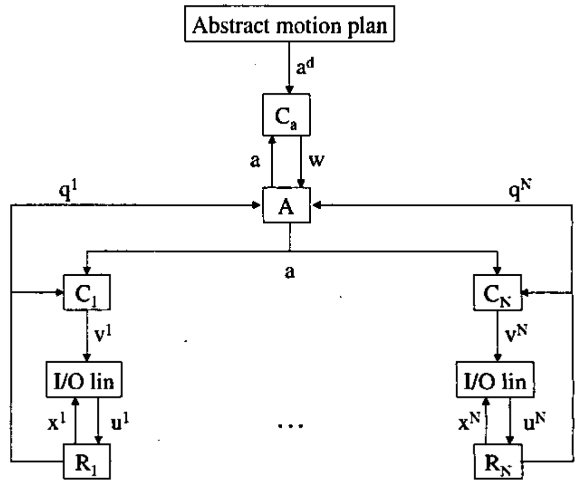

Figure 1: Overall control architecture

$$v = \sum_{i=1}^{n} \frac{d\phi_i^T}{d\phi_i d\phi_i^T} w_i \tag{16}$$

from which the decoupling of the control variables on A is obvious.

To limit the amount of inter - robot communication in the overall control scheme (requirement (v)), we propose an architecture where the control law u^i of a robot only depends on its own state x^i and the state of the low dimensional abstraction manifold $a \in A$. Since $u^i = A(x^i)v^i$ and $q^i = h(x^i)$, this is achieved if

$$v^i = \pi_i(v) = v^i(q^i, a) \tag{17}$$

Pictorially, the desired control architecture is given in Figure 1.

Finally, note that, from (14), it follows that the abstract state a is at rest ($\dot{a} = w = 0$) if and only if all the reference points q^i are at rest ($\dot{q} = v = 0$). This guarantees that each individual motion can be "seen" in the small dimensional manifold A and, therefore, can be "penalized" by control.

4 Control of spatial distribution

In this section we define a physically significant abstraction (11) with a product structure (12) as follows. For an arbitrary configuration $q \in Q$, the group part g of the abstract state a is defined by $g = (R, \mu) \in G = SE(2)$. Let

$$\mu = \frac{1}{N}\sum_{i=1}^{N} q_i \in \mathbb{R}^2 \tag{18}$$

Let the rotation $R \in SO(2)$ be parameterized by $\theta \in (-\pi/2, \pi/2)$. Then, by definition,

$$\theta = \tfrac{1}{2}\text{atan2}\Big(\sum_{i=1}^{N}(q_i-\mu)^T E_1 (q_i-\mu), \sum_{i=1}^{N}(q_i-\mu)^T E_2 (q_i-\mu)\Big) \quad (19)$$

In this paper we restrict our attention to a 2 - dimensional shape $s = [s_1, s_2]$ defined by

$$\begin{aligned} s_1 &= \tfrac{1}{2(N-1)}\sum_{i=1}^{N}(q_i-\mu)^T H_1 (q_i-\mu), \\ s_2 &= \tfrac{1}{2(N-1)}\sum_{i=1}^{N}(q_i-\mu)^T H_2 (q_i-\mu) \end{aligned} \quad (20)$$

where

$$E_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, E_3 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad (21)$$

and

$$H_1 = I_2 + R^2 E_2,\ H_2 = I_2 - R^2 E_2,\ H_3 = R^2 E_1 \quad (22)$$

It is obvious that the abstraction defined by (18), (19), and (20) has constant dimension $n = 5$ and is invariant to permutations of the robots (requirement (ii)). In [3], it is shown that $d\mu$, $d\theta$, ds_1, and ds_2 are orthogonal with respect to the Euclidean metric on Q. Therefore, each of the abstract variables can be controlled separately and requirement (iv) is satisfied. Then it makes sense to design separate controls $w = (w_\mu, w_\theta, w_{s_1}, w_{s_2})$ at a point $a = (\mu, \theta, s_1, s_2)$. Examples of feedback controllers on the small manifold A include stabilization to a point

$$\begin{aligned} \dot{\mu} = w_\mu = K_\mu(\mu^d - \mu), \quad \dot{\theta} = w_\theta = k_\theta(\theta^d - \theta) \\ \dot{s}_1 = w_{s_1} = k_{s_1}(s_1^d - s_1), \quad \dot{s}_2 = w_{s_2} = k_{s_2}(s_2^d - s_2) \end{aligned} \quad (23)$$

and trajectory tracking

$$\begin{aligned} \dot{\mu} &= w_\mu &&= K_\mu(\mu^d(t) - \mu(t)) + \dot{\mu}^d(t) \\ \dot{\theta} &= w_\theta &&= k_\theta(\theta^d(t) - \theta(t)) + \dot{\theta}^d(t) \\ \dot{s}_1 &= w_{s_1} &&= k_{s_1}(s_1^d(t) - s_1(t)) + \dot{s_1}^d(t) \\ \dot{s}_2 &= w_{s_2} &&= k_{s_2}(s_2^d(t) - s_2(t)) + \dot{s_2}^d(t) \end{aligned} \quad (24)$$

where $K_\mu \in \mathbb{R}^{2\times 2}$ is a positive definite matrix and k_θ, $k_{s_{1,2}} > 0$.

4.1 Significance

As shown in [3], there are two slightly different interpretations of the abstraction defined by (18), (19), and (20). Let

$$\Sigma = \frac{1}{N-1}\sum_{i=1}^{N}(q_i-\mu)(q_i-\mu)^T, \quad (25)$$

$$\Gamma = -(N-1)E_3 \Sigma E_3 \quad (26)$$

Equation (26) simply states that Γ is obtained from $(N-1)\Sigma$ by interchanging the diagonal elements and multiplying the extra - diagonal elements by -1. Therefore, Γ and $(N-1)\Sigma$ have the same eigenstructure.

First, μ and Γ in (18) and (26) can be seen as the centroid and inertia tensor of the system of particles q^i with respect to the centroid and orientation $\{W\}$. Let $\{M\}$ define a virtual frame with pose $g = (R, \mu)$ in $\{W\}$. The rotation equation (19) defines the orientation of the virtual frame so that the inertia tensor of the system of points in $\{M\}$ is diagonal. $(N-1)s_1$ and $(N-1)s_2$ are the eigenvalues of the tensor and are therefore measures of the spatial distribution of the reference points q^i along the axis of the virtual frame $\{M\}$. In [3] it is shown that the shape variables provide a bound for the region occupied by the robots: *An ensamble of N robots described by a 5 - dimensional abstract variable $a = (g, s) = (R, \mu, s_1, s_2)$ is enclosed in a rectangle centered at μ and rotated by $R \in SO(2)$ in the world frame $\{W\}$. The sides of the rectangle are given by $2\sqrt{(N-1)s_1}$ and $2\sqrt{(N-1)s_2}$.*

Alternatively, μ and Σ given by (18) and (25) can be interpreted as sample mean and covariance of a random variable with realizations q^i. If the random variable is known to be normally distributed, then, for a sufficiently large N, μ and Σ converge to the real parameters of the normal distribution. R is the rotation that diagonalizes the covariance and s_1, s_2 are the eigenvalues of the covariance matrix. This means that, for a large number of normally distributed reference points q^i, μ, R, s_1 and s_2 give the pose and semiaxes of a concentration ellipsoid.

Specifically, it is known that contours of constant probability p for normally distributed points in plane with mean μ and covariance Σ are ellipses described by

$$(x-\mu)^T\Sigma^{-1}(x-\mu) = c, \quad c = -2\ln(1-p) \quad (27)$$

The ellipse in (27), called *equipotential* or *concentration ellipse*, has the property that p percent of the points are inside it, and can be therefore used as a spanning region for our robots, under the assumption that they are normally distributed. Therefore we can make the following statement: *p percent of a large number N of normally distributed points described by a 5 - dimensional abstract variable $a = (g, s) = (R, \mu, s_1, s_2)$ is enclosed in an ellipse centered at μ, rotated by $R \in SO(2)$ in the world frame $\{W\}$ and with semiaxes $\sqrt{cs_1}$ and $\sqrt{cs_2}$, where c is given by (27).*

Even though the normal distribution assumption might seem very restrictive, we show in [3] that it is enough that the reference points q^i be normally distributed in the initial configuration. Our controls laws will preserve the normal distribution.

The abstraction based on the spanning rectangle has the advantage that it provides a rigurous bound for the region occupied by the robots and does not rely on any assumption on the distribution of the robots. The main disadvantage is that this estimate becomes too conservative when the number of robots is large. Indeed, the lengths of the sides of the

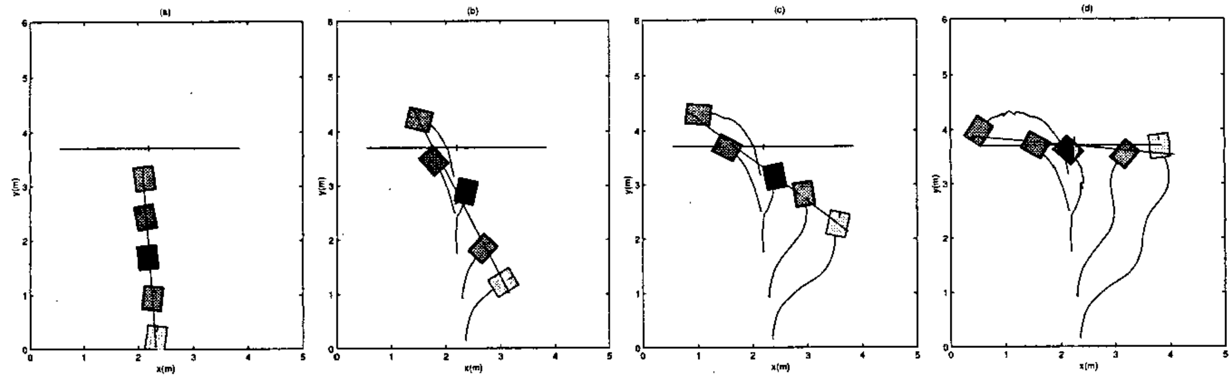

Figure 2: The position and orientation of the team is stabilized at desired values while shape is preserved

rectangle scale with $\sqrt{N-1}$, so for a large N the spanning rectangle might become very large, even though the robots might be grouped around the centroid μ.

On the other hand, the size of the concentration ellipsoid does not scale with the number of robots, which makes this approach very attractive for very large N. However, it has the disadvantage of assuming a normally distributed initial configuration of the team and does not provide a rigurous bound for the region occupied by the robots. Roughly speaking, $(1-p)N$ are left out of the p-ellipse. Increasing p will decrease the number of the robots which be outside but will also increase the size of the ellipsoid.

4.2 Individual control laws

In [3], it is shown that the submersion condition (i) is satisfied by the map ϕ defined by (18), (19), and (20) if and only if $s_1 \neq 0$ and $s_2 \neq 0$. These cases of zero shape physically correspond to degenerate situations when all the robots are on the Oy and Ox axis of the formation frame $\{M\}$, respectively. Excluding these degenerate cases, $d\phi d\phi^T$ is invertible and after some calculations, the projection (10) of (14) leads to the following velocity for the reference point P_i

$$v^i = \pi_i(v) = w_\mu + \frac{s_1-s_2}{s_1+s_2} H_3(q_i-\mu) w_\theta + \frac{1}{4s_1} H_1(q_i-\mu) w_{s_1} + \frac{1}{4s_2} H_2(q_i-\mu) w_{s_2} \quad (28)$$

The solution to Problem 1 is therefore given by u^i defined by (6) and (28)

Note that the overall control architecture implementing (28) fits the structure in Figure 1. Each robot i needs to implement controller C_i, which is only dependent on its own state x^i and the small dimensional abstract state a. Also, each robot has to send its output q^i to the abstract control system, which calculates and then broadcasts the updated abstract state. Therefore, robot i only has to broadcast its 2-dimensional output q^i and listen to a the 5-dimensional abstract state a, independent on the number of robots N.

In [3], it is shown that if control law (28) is applied to all the robots, then the set of points with cooedinates q^i in the plane undergoes an affine transformation. Any affine transformation is known to preserve collinearity, ratios of distances on lines, and parallelism. Therefore, control law (28) can be used for formations in which preserving properties like the ones mentioned above is important. Even more interesting, it is known that affine transformations preserve the normal distribution. This means that if the robots are initially normally distributed, by applying the control laws (28), they remain normally distributed. The 5 - dimensional abstract state, interpreted as sample mean μ and sample covariance Σ, gives us control over the pose, aspect ratio and size of the concentration ellipsoid as defined above.

4.3 Internal dynamics

Note that (23) or (24) only guarantee the desired behavior and therefore the boundness of the 5 - dimensional $a \in A$. Now the hardest problem, as in most input - output feedback linearization problems, is to prove the boudness of the internal dynamics. This usually implies a change of coordinates and explicit calculation of the zero dynamics. Fortunately, the boundness of $a \in A$ together with the definition of ϕ easily imply the boundness of each of q^i, $i = 1, \ldots, N$ [3]. Moreover, in the stabilization to a point case, (23), it can be proven that for any μ^d, θ^d, s_1^d, s_2^d, the closed loop system (28), (23) globally asymptotically converges on Q to the equilibrium manifold $\mu = \mu^d$, $\theta = \theta^d$, $s_1 = s_1^d$, $s_2 = s_2^d$. We conclude that the overall system is well behaved on Q. Moreover, the remaining 1 - dimensional internal dynamics of each robot can also be proved to be bounded. [4].

5 Experimental results

The experiments were performed using a team of five car-like robots. Each robot is equipped with its own processor and an omnidirectional camera. Communication among robots is needed to estimate the individual orientations x_3^i and relies on IEEE 802.11 networking. A calibrated overhead camera is used to localize the cartesian coordinates of the reference points q^i. A centralized computer calcu-

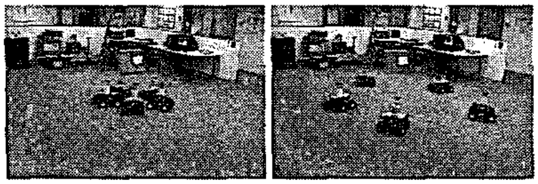

Figure 3: Initial and final configuration in an expansion maneuver - control of shape while pose in preserved

lates the 5 - dimensional team variable $a = (\mu, \theta, s_1, s_2)$ and broadcasts it back to the robots together with q^i from the overhead camera. Each robot has then complete information on its state x^i and the state a and can compute its own control u^i. All this computation is executed in approximately 15 Hz in the slowest computer. Note that the actual comunication architecture that we use in the experiments does not exactly fit the one we claim in Figure 1 because in our indoor setup the robots are not able to determine their own position and orientation.

5.1 Group control

In the first experiment we show how the pose of the team can be controlled while shape is preserved, illustrating the decoupling property of controllers (6), (28). The robots are initially "almost" alligned with the Oy axes of the world frame: $\mu = (2.2020, 1.6817)$, $\theta = -1.499\,rad$, $s_1 = 0.5417$, $s_2 = 1.6798e-4$. We use controllers (23) with $K_\mu = 4I_2$, $k_\theta = 4$, $k_{s_1} = k_{s_2} = 0$ to stabilize the team at $\mu^d = (2.2, 3.7)$, $\theta^d = 0$, and the shape, according to our theoretical results, should be preserved. The results are shown in Figure 2.

5.2 Shape control

The second experiment illustrates an expansion maneuver. Instead of plotting the experimental data, as in Section 5.1, we show the initial and final snaphots from the actual experiment in Figure 3. The robots were initially grouped in a small circle $s_1 = s_2 = 0.0738$ around $\mu = (2.4607, 2.6185)$. We again used stabilizing controllers (23) but this time with $K_\mu = 0$, $k_\theta = 0$, $k_{s_1} = k_{s_2} = 4$ to stabilize the team at $s_1^d = s_2^d = 0.6078$. The pose of the team, as predicted by our theoretical results, was preserved.

6 Conclusion and future work

We propose a control architecture for a large number of robots required to accomplish a task as a team. The cooperative mission is formulated in terms of cartesian coordinates of some reference points chosen on the robots. We propose an abstraction based on the definition of a map from the space of the reference points to a small dimensional abstraction manifold with a product structure of a Lie group and a shape space. The task to be accomplished by the team suggests a natural feedback control system on the abstraction manifold. We focus on car - like robots and show that the group and shape variables can be controlled separately. The individual control laws which are mapped to the desired behavior of the team can be realized by feedback depending only on the robots' current state and the small dimensional state on the abstraction manifold. Future work will be directed towards incorporating more shape variables, accomodating other types of under-actuation constraints, extending the results to 3-D environments, and implementing the obtained control architectures in our blimp - car outdoor experimental platform.

Acknowledgments

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