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Controlling a Network of Signalized Intersections From Temporal Logical Specifications

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***Abstract*— We propose a framework for generating a control policy for a traffic network of signalized intersections to accomplish control objectives expressed in linear temporal logic. Traffic management indeed calls for a rich class of objectives and offers a novel domain for these formal methods tools. We show that traffic networks possess structural properties that allow significant reduction in the time required to compute a finite state abstraction. We further extend our approach to a probabilistic framework by modeling the traffic dynamics as a Markov Decision Process.**

I. Introduction

Control of networks of signalized intersections has received considerable attention in recent decades; see [1] for a review. Many existing strategies focus on limited objectives such as maximizing throughput [2] or maintaining stability of network queues [3]. However, efficient traffic management often calls for a range of objectives beyond those mentioned above. In this work, we consider control objectives expressible using *linear temporal logic (LTL)* [4], [5]. For example, LTL formula allows objectives such as "infinitely often, the queue length on road ℓ should reach 0" and "anytime link ℓ becomes congested, it eventually becomes uncongested."

In this paper, we leverage recent results on control synthesis from LTL specifications such as [6]–[11] to design signal control policies for traffic networks. We model the traffic network as a network of queues [2], [3] with link capacities. The result is a piecewise-affine (PWA) dynamical model, and we describe a method for obtaining a finite state abstraction of the dynamics using polyhedral operations as proposed in [7]. A related approach to control of freeway traffic, which only considers ramp metering, is proposed in [12]. Additionally, [13] suggests using mode sequences as abstract states rather then partitions of the state space and applies the technique to a simple model of traffic flow with no link capacities or exogenous disturbance input.

Next, due to the expense of performing the required polyhedral operations in high dimensions, we propose an approximate finite state abstraction which can be computed much more efficiently. This efficiency is due to properties of the traffic flow model which allow efficient computation of bounds on the one-step reachable states of a given link. We then suggest a method for extending these results to a probabilistic framework. Our work [14] does not present this extension or a method based on polyhedral operations.

This paper is organized as follows: Section II describes the traffic network dynamical model. Section III presents the LTL-based approach to control synthesis based on a computationally efficient method of constructing a finite state representation of the network dynamics. We extend these results to a probabilistic formulation in Section IV. A case study is presented in Section V, and we provide concluding remarks in Section VI.

II. Signalized Network Traffic Model

A signalized traffic network consists of a set $\mathcal{L}$ of *links* and a set $\mathcal{V}$ of *signalized intersections* or *nodes*. For $\ell \in \mathcal{L}$, let $\sigma(\ell) \in \mathcal{V}$ denote the head node of link ℓ and let $\tau(\ell) \in \mathcal{V} \cup \emptyset$ denote the tail node of link ℓ. A link ℓ with $\tau(\ell) = \emptyset$ serves as an entry-point into the network, and we assume $\sigma(\ell) \neq \tau(\ell)$ for all $\ell \in \mathcal{L}$ (*i.e.*, no self-loops). Link $k \neq \ell$ is *upstream* of link ℓ if $\sigma(k) = \tau(\ell)$, *downstream* of link ℓ if $\tau(k) = \sigma(\ell)$, and *adjacent to* link ℓ if $\tau(k) = \tau(\ell)$. Roads exiting the traffic network are not modeled. For each $v \in \mathcal{V}$, define

$$\mathcal{L}_v^{\text{in}} = \{\ell : \sigma(\ell) = v\}, \qquad \mathcal{L}_v^{\text{out}} = \{\ell : \tau(\ell) = v\}. \tag{1}$$

Each link $\ell \in \mathcal{L}$ possesses a queue $x_\ell[t] \in [0, x_\ell^{\text{cap}}]$ representing the number of vehicles on link ℓ at time step $t \in \mathbb{N} \triangleq \{0, 1, 2, \ldots\}$ where x_ℓ^{cap} is the capacity of link ℓ. We allow x_ℓ to be a continuous quantity, thus adopting a fluid-like model of traffic flow. Let $\mathcal{X} = \prod_{\ell \in \mathcal{L}} [0, x_\ell^{\text{cap}}]$.

Movement of vehicles among link queues is governed by mass-conservation laws and the state of the signalized intersections. When a link is *actuated*, a maximum of c_ℓ vehicles are allowed to flow from link ℓ to links $\mathcal{L}_{\sigma(\ell)}^{\text{out}}$ per time step where c_ℓ is the known *saturation flow* for link ℓ. To simplify notation, we assume each intersection $v \in \mathcal{V}$ has two possible states actuating either "East-West" (EW) incoming links or "North-South" (NS) incoming links[1].

To make this precise, we partition $\mathcal{L}$ into EW links and NS links, denoted by $\mathcal{L}^{\text{EW}}$ and $\mathcal{L}^{\text{NS}}$, respectively, so that $\mathcal{L} = \mathcal{L}^{\text{EW}} \cup \mathcal{L}^{\text{NS}}$. We define the signal variable $s_v \in \{0, 1\}$ as follows for each $v \in \mathcal{V}$:

$$s_v = \begin{cases} 1 & \text{if links } \mathcal{L}_v^{\text{in}} \cap \mathcal{L}^{\text{EW}} \text{ are actuated} \\ 0 & \text{if links } \mathcal{L}_v^{\text{in}} \cap \mathcal{L}^{\text{NS}} \text{ are actuated.} \end{cases} \tag{2}$$

When a link ℓ is actuated, the *turn ratio* $\beta_{\ell k}$ denotes the fraction of vehicles exiting link ℓ that are routed to link k.

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[1] We can easily generalize to signal variables with more than two states and general network topologies at the cost of more complex notation.

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It follows that $\beta_{\ell k} \neq 0$ only if $\sigma(\ell) = \tau(k)$ and

$$\sum_{k \in \mathcal{L}^{\text{out}}_{\sigma(\ell)}} \beta_{\ell k} \leq 1. \tag{3}$$

Strict inequality in (3) implies that a fraction of vehicles on link ℓ are routed off the network via unmodeled roads that exit the network. Even when a link is actuated, traffic flow can occur only if there is available capacity downstream. To this end, the supply ratio $\alpha_{\ell k}$ denotes the fraction of link k's capacity available to link ℓ, that is, link ℓ may only send $\alpha_{\ell k}(x^{\text{cap}}_k - x_k[t])$ vehicles to link k in time period t. Since only incoming EW or NS links are actuated in each time step, it follows that, for all $k \in \mathcal{L}$,

$$\sum_{\ell \in \mathcal{L}^{\text{in}}_{\tau(k)} \cap \mathcal{L}^{\text{EW}}} \alpha_{\ell k} = \sum_{\ell \in \mathcal{L}^{\text{in}}_{\tau(k)} \cap \mathcal{L}^{\text{NS}}} \alpha_{\ell k} = 1. \tag{4}$$

We are now in a position to define the dynamics of the link queues. We first define the following:

$$\mathcal{L}^{\text{down}}_\ell = \mathcal{L}^{\text{out}}_{\sigma(\ell)} \cup \{\ell\} \qquad \mathcal{L}^{\text{up}}_\ell = \mathcal{L}^{\text{in}}_{\tau(\ell)} \tag{5}$$

$$\mathcal{L}^{\text{adj}}_\ell = \mathcal{L}^{\text{out}}_{\tau(\ell)} \backslash \{\ell\} \qquad \mathcal{L}^{\text{loc}}_\ell = \mathcal{L}^{\text{down}}_\ell \cup \mathcal{L}^{\text{up}}_\ell \cup \mathcal{L}^{\text{adj}}_\ell \tag{6}$$

so that $\mathcal{L}^{\text{down}}_\ell$ includes link ℓ and the links downstream of link ℓ, $\mathcal{L}^{\text{up}}_\ell$ and $\mathcal{L}^{\text{adj}}_\ell$ are links upstream and adjacent to ℓ, respectively. As we will see subsequently, the flow of vehicles out of link ℓ is only a function of the state of links in $\mathcal{L}^{\text{down}}_\ell$, and the update of link ℓ's state is only a function of links in $\mathcal{L}^{\text{loc}}_\ell$, that is, links "local" to link ℓ.

Let $\mathbf{x}[t] = \{x_\ell[t]\}_{\ell \in \mathcal{L}}$, $\mathbf{s}[t] = \{s_v[t]\}_{v \in \mathcal{V}}$, $\mathbf{x}^{\text{down}}_\ell = \{x_k\}_{k \in \mathcal{L}^{\text{down}}_\ell}$, and $\mathbf{x}^{\text{loc}}_\ell = \{x_k\}_{k \in \mathcal{L}^{\text{loc}}_\ell}$. We define the outflow of link ℓ for all $\ell \in \mathcal{L}$ as follows:

$$f^{\text{out}}_\ell(\mathbf{x}^{\text{down}}_\ell, s_{\sigma(\ell)}) = \begin{cases} s_{\sigma(\ell)} \cdot \phi_\ell(\mathbf{x}^{\text{down}}_\ell[t]) & \text{if } \ell \in \mathcal{L}^{\text{EW}} \\ (1 - s_{\sigma(\ell)}) \cdot \phi_\ell(\mathbf{x}^{\text{down}}_\ell[t]) & \text{if } \ell \in \mathcal{L}^{\text{NS}}, \end{cases} \tag{7}$$

$$\phi_\ell(\mathbf{x}^{\text{down}}_\ell[t]) = \min\left\{x_\ell[t], c_\ell, \min_{\substack{k \text{ s.t.} \\ \beta_{\ell k} \neq 0}} \left\{\frac{\alpha_{\ell k}}{\beta_{\ell k}}(x^{\text{cap}}_k - x_k[t])\right\}\right\}. \tag{8}$$

The number of vehicles in each link's queue then evolves according to the mass conservation equation

$$x_\ell[t+1] = F_\ell(\mathbf{x}^{\text{loc}}_\ell[t], \mathbf{s}^{\text{loc}}[t], d_\ell[t]) \tag{9}$$

$$\triangleq x_\ell[t] - f^{\text{out}}_\ell(\mathbf{x}^{\text{down}}_\ell[t], s_{\sigma(\ell)}[t]) + \sum_{j \in \mathcal{L}^{\text{up}}_\ell} \beta_{j\ell} f^{\text{out}}_j(\mathbf{x}^{\text{down}}_j[t], s_{\sigma(j)}[t]) + d_\ell[t] \tag{10}$$

where $d_\ell[t]$ is the number of vehicles that exogenously enters the queue on link ℓ in time step t, $\mathbf{d} = \{d_\ell[t]\}_{\ell \in \mathcal{L}}$, and $\mathbf{s}^{\text{loc}}_\ell = \{s_{\sigma(\ell)}, s_{\tau(\ell)}\}$, that is, $\mathbf{s}^{\text{loc}}_\ell$ is the state of the signals that are "local" to link ℓ (if $\tau(\ell) = \emptyset$, then we take $\mathbf{s}^{\text{loc}}_\ell = \{s_{\sigma(\ell)}\}$). We assume there exists $\mathcal{D} \subset \mathbb{R}^{|\mathcal{L}|}$ such that

$$\mathbf{d}[t] \in \mathcal{D} \quad \forall t. \tag{11}$$

We let $F(\mathbf{x}, \mathbf{s}, \mathbf{d}) = \{F_\ell(\mathbf{x}^{\text{loc}}, \mathbf{s}^{\text{loc}}, d_\ell)\}_{\ell \in \mathcal{L}}$ so that

$$\mathbf{x}[t+1] = F(\mathbf{x}[t], \mathbf{s}[t], \mathbf{d}[t]). \tag{12}$$

We then have $F(\mathbf{x}, \mathbf{s}, \mathbf{d}) : \mathbb{R}^{|\mathcal{L}|} \times \{0,1\}^{|\mathcal{V}|} \times \mathcal{D} \to \mathbb{R}^{|\mathcal{L}|}$. Finally, we define the set $\mathcal{L}^{\text{in}}_{v,\mathbf{s}}$ to be the set of links actuated by signal $\mathbf{s} \in \{0,1\}^{|\mathcal{V}|}$ at intersection v, that is,

$$\mathcal{L}^{\text{in}}_{v,\mathbf{s}} = \begin{cases} \mathcal{L}^{\text{in}}_v \cap \mathcal{L}^{\text{EW}} & \text{if } s_v = 0 \\ \mathcal{L}^{\text{in}}_v \cap \mathcal{L}^{\text{NS}} & \text{if } s_v = 1. \end{cases} \tag{13}$$

III. Controller Synthesis from Linear Temporal Logic Specifications

We now turn to the main objective this paper, namely, synthesizing a signal control strategy such that the resulting traffic dynamics and signal sequence satisfies a control objective expressed using *linear temporal logic (LTL)*. We first define and motivate the need for the rich class of control objectives expressible in LTL in the context of traffic networks. We then propose a control synthesis approach which relies on a finite state representation of the traffic dynamics.

A. LTL Specifications for Traffic Networks

LTL formulae describe properties of trajectories of the traffic network and are generated inductively using the Boolean operators $\vee$ (disjunction), $\wedge$ (conjunction), $\neg$ (negation), and the temporal operators $\bigcirc$ (next) and U (until). Formally, such formulae are expressed over the states of the finite state representation constructed below. We use LTL formulae to describe desired behaviors of the traffic network, which allows very rich control specifications that include derived logical and temporal operators such as $\rightarrow$ (implication), $\square$ (always), and $\diamond$ (eventually), see [4], [5].

Examples of LTL formulae representing desired control objectives relevant to traffic networks include those from the Introduction as well as:

- $\varphi = \diamond\square(x_\ell \leq C)$ for some C
  "Eventually, link ℓ has less than C vehicles for all future time"
- $\varphi = \square((s_{v_1} = 1) \rightarrow \bigcirc(s_{v_2} = 1))$
  "Whenever signal v_1 actuates EW traffic, signal v_2 must actuate EW traffic in the next time step"
- $\varphi = \square((x_{\ell_1} \geq C_1) \rightarrow (x_{\ell_2} \geq C_2))$
  "The number of vehicles on link ℓ_1 exceeds C_1 only if the number of vehicles exceeds C_2 on link ℓ_2."

To generate control strategies for the traffic network that guarantee satisfaction of a LTL formula, we first construct a finite state representation, or *abstraction*, of the model defined in Section II. In the next section, we present an abstraction approach based on polyhedral operations, and we then show that a modified abstraction can be efficiently constructed by exploiting the sparsity and the dynamical properties of the traffic network.

B. Finite State Representation

We begin by defining a *quotient transition system* obtained via a rectangular partition of the state space that over-approximates the traffic dynamics. In particular, we consider partitioning each interval $[0, x^{\text{cap}}_\ell]$ into the set of intervals

$$\{[0, x^1_\ell], (x^1_\ell, x^2_\ell], \ldots, (x^{N_\ell - 2}_\ell, x^{N_\ell - 1}_\ell], (x^{N_\ell - 1}_\ell, x^{N_\ell}_\ell]\} \tag{14}$$

where $x_\ell^i < x_\ell^{i+1}$ for all i and $x_\ell^{N_\ell} = x_\ell^{\text{cap}}$. By convention, we let $x_\ell^0 = 0$. For $X_\ell \in \{1, \dots, N_\ell\}$, let

$$[\![X_\ell]\!] = \begin{cases} [x_\ell^{X_\ell - 1}, x_\ell^{X_\ell}] & \text{if } X_\ell = 1 \\ (x_\ell^{X_\ell - 1}, x_\ell^{X_\ell}] & \text{if } X_\ell > 1, \end{cases} \tag{15}$$

i.e., $[\![\cdot]\!]$ gives the interval corresponding to the discrete state X_ℓ. For $\mathbf{X} = \{X_\ell\}_{\ell \in \mathcal{L}}$, we let

$$[\![\mathbf{X}]\!] = \prod_{\ell \in \mathcal{L}} [\![X_\ell]\!] \subset \mathcal{X}. \tag{16}$$

Definition 1 (Quotient Transition System)**.** *Given an interval partition as in* (14) *for each link $\ell \in \mathcal{L}$, a finite, nondeterministic quotient transition system of the traffic model is defined as the tuple $T = (\mathbb{X}, \mathbb{S}, \rightarrow)$ where*

- $\mathbb{X} = \prod_{\ell \in \mathcal{L}} \{1, \dots, N_\ell\}$ *is the set of states,*
- $\mathbb{S} = \{0, 1\}^{|\mathcal{V}|}$ *is the set of controls,*
- $\rightarrow \subseteq \mathbb{X} \times \mathbb{S} \times \mathbb{X}$ *is the set of transitions given by* $(\mathbf{X}, \mathbf{s}, \mathbf{X}') \in \rightarrow$ *iff there exists* $\mathbf{d} \in \mathcal{D}$ *such that*

$$\exists \mathbf{x} \in [\![\mathbf{X}]\!], \exists \mathbf{x}' \in [\![\mathbf{X}']\!] \text{ such that } \mathbf{x}' = F(\mathbf{x}, \mathbf{s}, \mathbf{d}). \tag{17}$$

In words, a discrete state $\mathbf{X}$ transitions to $\mathbf{X}'$ under signal input $\mathbf{s} \in \mathbb{S}$ if and only if for some disturbance $\mathbf{d} \in \mathcal{D}$, there exists continuous states $\mathbf{x} \in [\![\mathbf{X}]\!]$ and $\mathbf{x}' \in [\![\mathbf{X}']\!]$ such that $\mathbf{x}$ may transition to $\mathbf{x}'$ under the signaling input $\mathbf{s}$.

All possible trajectories of the traffic network dynamics are represented in T since T is an *over-approximation* of the traffic network model; due to this approximation, there may exist *spurious* executions of the quotient system that do not correspond to any trajectory of the original traffic network. Furthermore, T is nondeterministic due to these spurious trajectories and due to the disturbance taking values within a set. While the over-approximation implies possible conservatism in our approach [5], it does not affect soundness of the synthesis algorithm. In particular, we synthesize a controller that guarantees satisfaction of the control objective for all executions of the quotient system, which encompasses all possible trajectories of the original system.

We now discuss a method for obtaining the quotient transition system $T = (\mathbb{X}, \mathbb{S}, \rightarrow)$ from the dynamics presented in Section III-B. We observe that (7)–(11) result in dynamics that are *piecewise affine*, that is, there exists a set of polytopes $\{\mathcal{X}_p\}_{p \in \mathcal{P}}$ for some index set $\mathcal{P}$ such that $\cup_{p \in \mathcal{P}} \mathcal{X}_p = \mathcal{X}$ and $\text{int}(\mathcal{X}_p) \cap \text{int}(\mathcal{X}_q) = \emptyset$ for all $p \neq q$, and such that for each $p \in \mathcal{P}$, we have

$$F(\mathbf{x}, \mathbf{s}, \mathbf{d}) = A_{p,\mathbf{s}} \mathbf{x} + b_{p,\mathbf{s}} + \mathbf{d} \qquad \forall \mathbf{x} \in \mathcal{X}_p \tag{18}$$

for some $A_{p,\mathbf{s}} \in \mathbb{R}^{|\mathcal{L}| \times |\mathcal{L}|}$, $b_{p,\mathbf{s}} \in \mathbb{R}^{|\mathcal{L}|}$. In other words, the traffic dynamics are affine in each polyhedral partition. The polytopes arise from the $\min\{\cdot\}$ functions in (8).

For $\mathbf{X} \in \mathbb{X}$, let $\{\mathcal{X}_p^{\mathbf{X}}\}_{p \in \mathcal{P}}$ be the partition of $[\![\mathbf{X}]\!]$ with respect to $\mathcal{P}$, that is, $\mathcal{X}_p^{\mathbf{X}} = [\![\mathbf{X}]\!] \cap \mathcal{X}_p$ (in general, many $\mathcal{X}_p^{\mathbf{X}}$ will be empty).

The set of states of system (12) that are reachable from a set $Y \subset \mathcal{X}$ under the control signal $\mathbf{s}$ is denoted by the $\texttt{Post}$ operator and given by

$$\texttt{Post}(Y, \mathbf{s}) = \{\mathbf{x}' = F(\mathbf{x}, \mathbf{s}, \mathbf{d}) \mid \mathbf{x} \in Y, \mathbf{d} \in \mathcal{D}\}. \tag{19}$$

It follows that for $\mathbf{X} \in \mathbb{X}$,

$$\texttt{Post}([\![\mathbf{X}]\!], \mathbf{s}) = \bigcup_{p \in \mathcal{P}} \texttt{Post}(\mathcal{X}_p^{\mathbf{X}}, \mathbf{s}). \tag{20}$$

If $\mathcal{D}$ is assumed to be a polytope, then $\texttt{Post}(\mathcal{X}_p^{\mathbf{X}}, \mathbf{s})$ is computed through basic polyhedral operations since each $\mathcal{X}_p^{\mathbf{X}}$ is a polytope and the dynamics are affine in $\mathcal{X}_p^{\mathbf{X}}$ under the control signal $\mathbf{s}$ as in (18), see [7] for details.

Let $\texttt{Post}_T(\mathbf{X}, \mathbf{s}) \subset \mathbb{X}$ be the set of discrete states that the quotient transition system may transition to under signal $\mathbf{s}$ when in discrete state $\mathbf{X}$, that is

$$\texttt{Post}_T(\mathbf{X}, \mathbf{s}) = \{\mathbf{X}' \mid (\mathbf{X}, \mathbf{s}, \mathbf{X}') \in \rightarrow\}. \tag{21}$$

According to Definition 1, we have

$$\texttt{Post}_T(\mathbf{X}, \mathbf{s}) = \{\mathbf{X}' \mid \texttt{Post}([\![\mathbf{X}]\!], \mathbf{s}) \cap [\![\mathbf{X}']\!] \neq \emptyset\}. \tag{22}$$

Therefore, the quotient transition system can be constructed by performing a set of polyhedral operations. However, these operations scale exponentially in the number of links. Next, we propose constructing an approximation of T that does not require polyhedral operations.

C. Approximate Quotient Transition System

We now introduce an approximate quotient system which can be constructed much more efficiently than the system proposed in Definition 1.

Definition 2 (Approximate Quotient Transition System)**.** *Given an interval partition as in* (14) *for each $\ell \in \mathcal{L}$, an approximate finite quotient system of the traffic dynamics is defined as the tuple $T' = (\mathbb{X}, \mathbb{S}, \rightarrow')$ where*

- $\mathbb{X} = \prod_{\ell \in \mathcal{L}} \{1, \dots, N_\ell\}$ *is the set of states,*
- $\mathbb{S} = \{0, 1\}^{|\mathcal{V}|}$ *is the set of controls,*
- $\rightarrow' \subseteq \mathbb{X} \times \mathbb{S} \times \mathbb{X}$ *is the set of transitions given by* $(\mathbf{X}, \mathbf{s}, \mathbf{X}') \in \rightarrow'$ *iff there exists* $\mathbf{d} \in \mathcal{D}$ *such that*

$$\forall \ell \in \mathcal{L}, \; \exists \mathbf{x} \in [\![\mathbf{X}]\!], \exists x'_\ell \in [\![X'_\ell]\!] \text{ such that } x'_\ell = F_\ell(\mathbf{x}, \mathbf{s}, d_\ell). \tag{23}$$

In words, a discrete state $\mathbf{X}$ transitions to $\mathbf{X}' = \{X'_\ell\}_{\ell \in \mathcal{L}}$ under signal input $\mathbf{s} \in \mathbb{S}$ if and only if there exists $\mathbf{d} \in \mathcal{D}$ and, for each link $\ell \in \mathcal{L}$, there exists $x'_\ell \in [\![X'_\ell]\!]$ and $\mathbf{x} \in [\![\mathbf{X}]\!]$ such that $x'_\ell = F_\ell(\mathbf{x}^{loc}, \mathbf{s}^{loc}, d_\ell)$.

The difference between Definitions 1 and 2 is that in verifying (23), a different $\mathbf{x} \in [\![\mathbf{X}]\!]$ may be chosen for each ℓ, whereas (17) must hold for a particular $\mathbf{x} \in [\![\mathbf{X}]\!]$. This difference allows us to exploit the structure and sparsity of traffic network dynamics to efficiently compute T'. Furthermore, like T, the transition system T' is an over-approximation of the traffic network model.

D. Efficient Computation of Approximate Quotient System

We now present an algorithm for efficiently computing T' when $\mathcal{D}$ is a union of hyperrectangles where computation of $\{\mathbf{X}' \mid (\mathbf{X}, \mathbf{s}, \mathbf{X}') \in \rightarrow'\}$ only requires evaluating F_ℓ at two corners of the hyperrectangle $[\![\mathbf{X}]\!]$ for each ℓ.

We first assume $\mathcal{D} = \bigcup_{i=1}^{n_\mathcal{D}} \mathcal{D}_i$ for some $n_\mathcal{D} \in \mathbb{N}$ where $\mathcal{D}_i = \prod_{\ell \in \mathcal{L}} [\underline{d}_i^\ell, \overline{d}_i^\ell]$ for all $i \in \{1, \dots, n_\mathcal{D}\}$

where $\underline{d}_i^\ell \leq \overline{d}_i^\ell$ for all $\ell \in \mathcal{L}$. Let $\texttt{Post}_{T'}(\mathbf{X}, \mathbf{s}) = \{\mathbf{X}' \mid (\mathbf{X}, \mathbf{s}, \mathbf{X}') \in \rightarrow'\}$ and let $\texttt{Post}_{T',\mathcal{Y}}(\mathbf{X}, \mathbf{s}) = \{\mathbf{X}' \mid (\mathbf{X}, \mathbf{s}, \mathbf{X}') \in \rightarrow'$ when $\mathcal{D}$ is replaced with $\mathcal{Y}$ in (23)$\}$. Then $\texttt{Post}_{T'}(\mathbf{X}, \mathbf{s}) = \texttt{Post}_{T',\mathcal{D}}(\mathbf{X}, \mathbf{s}) = \bigcup_{i=1}^{n_\mathcal{D}} \texttt{Post}_{T',\mathcal{D}_i}(\mathbf{X}, \mathbf{s})$, and we thus focus on computation of $\texttt{Post}_{T',\mathcal{D}_i}(\mathbf{X}, \mathbf{s})$. For given $\mathbf{X} \in \mathbb{X}$, $\mathbf{s} \in \mathbb{S}$, and $\mathcal{D}_i$, let

$$\underline{x}'_\ell = \min_{\mathbf{x} \in [\![\mathbf{X}]\!], d_\ell \in [\underline{d}_i^\ell, \overline{d}_i^\ell]} F_\ell(\mathbf{x}_\ell^{\text{loc}}, \mathbf{s}_\ell^{\text{loc}}, d_\ell) \tag{24}$$

$$\overline{x}'_\ell = \max_{\mathbf{x} \in [\![\mathbf{X}]\!], d_\ell \in [\underline{d}_i^\ell, \overline{d}_i^\ell]} F_\ell(\mathbf{x}_\ell^{\text{loc}}, \mathbf{s}_\ell^{\text{loc}}, d_\ell). \tag{25}$$

Proposition 1. *Given $\mathbf{X} \in \mathbb{X}$, $\mathbf{s} \in \mathbb{S}$, $\mathcal{D}_i$ as above, and $\{\underline{x}'_\ell, \overline{x}'_\ell\}_{\ell \in \mathcal{L}}$ as defined in* (24)–(25)*. We have* $\texttt{Post}_{T',\mathcal{D}_i}(\mathbf{X}, \mathbf{s}) = \{\mathbf{X}' = \{X'_\ell\}_{\ell \in \mathcal{L}} \mid [\![X'_\ell]\!] \cap [\underline{x}'_\ell, \overline{x}'_\ell] \neq \emptyset \ \forall \ell \in \mathcal{L}\}$.

Proposition 1 follow straightforwardly from the rectangular form of $\mathcal{D}_i$ and the definition of $\rightarrow'$ in Definition 1.

The computational advantage of Definition 2 over Definition 1 comes from the fact that the right hand sides of (24) and (25) can be computed efficiently. We first make the following technical assumption which is not particularly restrictive and can always be satisfied in traffic networks when a short enough time step is considered:

Assumption 1. *For all $\ell \in \mathcal{L}$,*

$$c_\ell \leq x_\ell^{cap} - \frac{\beta_{k\ell}}{\alpha_{k\ell}} c_k \quad \forall k \in \mathcal{L}_\ell^{up}. \tag{26}$$

Assumption 1 is a sufficient condition for ensuring that a link cannot completely clear its queue in one time step while simultaneously restricting flow from an upstream link, which is required for the following proposition.

Proposition 2. *Given $\mathbf{X} = \{X_\ell\}_{\ell \in \mathcal{L}} \in \mathbb{X}$ where $X_\ell = [x_\ell^{X_\ell - 1}, x_\ell^{X_\ell}]$ or $X_\ell = (x_\ell^{X_\ell - 1}, x_\ell^{X_\ell}]$ as in* (15)*. For given $\mathbf{s} \in \mathbb{S}$ and $\mathbf{d} \in \mathcal{D}_i$, let*

$$\underline{\mathbf{x}}_\ell^{loc} = \bigcup_{\substack{k \in \\ \mathcal{L}_\ell^{down} \cup \mathcal{L}_\ell^{up}}} \{x_k^{X_k - 1}\} \cup \bigcup_{k \in \mathcal{L}_\ell^{adj}} \{x_k^{X_k}\} \tag{27}$$

$$\overline{\mathbf{x}}_\ell^{loc} = \bigcup_{\substack{k \in \\ \mathcal{L}_\ell^{down} \cup \mathcal{L}_\ell^{up}}} \{x_k^{X_k}\} \cup \bigcup_{k \in \mathcal{L}_\ell^{adj}} \{x_k^{X_k - 1}\}. \tag{28}$$

Then

$$\min_{\mathbf{x} \in [\![\mathbf{X}]\!], d_\ell \in [\underline{d}_i^\ell, \overline{d}_i^\ell]} F_\ell(\mathbf{x}_\ell^{loc}, \mathbf{s}_\ell^{loc}, d_\ell) = F_\ell(\underline{\mathbf{x}}_\ell^{loc}, \mathbf{s}_\ell^{loc}, \underline{d}_i^\ell) \tag{29}$$

$$\max_{\mathbf{x} \in [\![\mathbf{X}]\!], d_\ell \in [\underline{d}_i^\ell, \overline{d}_i^\ell]} F_\ell(\mathbf{x}_\ell^{loc}, \mathbf{s}_\ell^{loc}, d_\ell) = F_\ell(\overline{\mathbf{x}}_\ell^{loc}, \mathbf{s}_\ell^{loc}, \overline{d}_i^\ell). \tag{30}$$

We provide the following proof sketch: Observe that $\underline{\mathbf{x}}_\ell^{\text{loc}}$ is the collection of lower bounds for link ℓ and the links that are upstream and downstream of link ℓ, and the upper bounds for the links adjacent to link ℓ. Likewise, $\overline{\mathbf{x}}_\ell^{\text{loc}}$ is the collection of upper bounds for link ℓ and the links that are upstream and downstream of link ℓ, and the lower bounds for the links adjacent to link ℓ. Furthermore, the structure of traffic dynamics renders $F_\ell(\mathbf{x}_\ell^{\text{loc}}, \mathbf{s}^{\text{loc}}, d_\ell)$ monotonically increasing in x_k for $k \in (\mathcal{L}_\ell^{\text{down}} \cup \mathcal{L}_\ell^{\text{up}}) \backslash \{\ell\}$ and d_ℓ, and monotonically decreasing in x_k for $k \in \mathcal{L}_\ell^{\text{adj}}$. Thus $\underline{\mathbf{x}}_\ell^{\text{loc}}, \underline{d}_i^\ell$ are the conditions ensuring that x'_ℓ achieves the minimum possible under the constraint $\mathbf{x} \in [\![\mathbf{X}]\!]$ and $\mathbf{d} \in \mathcal{D}_i$, and similarly $\overline{\mathbf{x}}_\ell^{\text{loc}}, \overline{d}_i^\ell$ are the conditions ensuring that x'_ℓ achieves the maximum possible under the same constraints.

Combining Propositions 1 and 2, we have the following:

Corollary 1. *Given $\mathbf{X} \in \mathbb{X}$, $\mathbf{s} \in \mathbb{S}$, $\mathcal{D}_i$ as above, and $\underline{\mathbf{x}}_\ell^{loc}, \overline{\mathbf{x}}_\ell^{loc}$ for each $\ell \in \mathcal{L}$ given by* (29)–(30)*, we have*

$$\texttt{Post}_{T',\mathcal{D}_i}(\mathbf{X}, \mathbf{s}) = \{\mathbf{X}' = \{X'_\ell\}_{\ell \in \mathcal{L}} \mid [\![X'_\ell]\!] \cap [\underline{x}'_\ell, \overline{x}'_\ell] \neq \emptyset \ \forall \ell \in \mathcal{L}\} \tag{31}$$

where $\underline{x}'_\ell = F_\ell(\underline{\mathbf{x}}_\ell^{loc}, \mathbf{s}_\ell^{loc}, \underline{d}_i^\ell)$ and $\overline{x}'_\ell = F_\ell(\overline{\mathbf{x}}_\ell^{loc}, \mathbf{s}_\ell^{loc}, \overline{d}_i^\ell)$.

Corollary 1 exploits the structure and sparsity of the traffic dynamics and provides the key for efficient computation of the approximate quotient system T'. Indeed, sparsity allows consideration of only links and signals that are "local" to a link, and the structure of the dynamics is the foundation for Proposition 2 and, in turn, efficient computation of $\underline{x}'_\ell$ and $\overline{x}'_\ell$. Note that obtaining $\underline{x}'_\ell$ and $\overline{x}'_\ell$ requires computing F_ℓ at two points for each $\ell \in \mathcal{L}$, and that the two points $\underline{\mathbf{x}}_\ell^{\text{loc}}$ and $\overline{\mathbf{x}}_\ell^{\text{loc}}$ are easily obtained from $\mathbf{X} \in \mathbb{X}$ using (27)–(28). For disturbance set $\mathcal{D} = \bigcup_{i=1}^{n_\mathcal{D}} \mathcal{D}_i$, we compute $\texttt{Post}_{T',\mathcal{D}_i}$ for each $\mathcal{D}_i$ and combine the resulting transitions. It follows that computing $\{\mathbf{X}' \mid (\mathbf{X}, \mathbf{s}, \mathbf{X}') \in \rightarrow'\}$ for some $\mathbf{X}$ and $\mathbf{s}$ scales linearly with the dimension of the continuous state space, *i.e.*, the number of links in the network.

E. Controller Synthesis

We omit the details of how a control strategy is synthesized from the nondeterministic transition system T' for a given LTL control objective as this is well-documented elsewhere in the literature, see *e.g.* [7], [15]. Instead, we summarize the main steps of this synthesis as follows: from the LTL control objective, we obtain a deterministic Rabin automaton that accepts all and only trajectories that satisfy the LTL specification using off-the-shelf software. We then construct the synchronous product of the Rabin automaton and T', resulting in a nondeterministic Rabin automaton from which a control strategy is found by playing a Rabin game [15]. The result is a control strategy along with a set of initial conditions from which trajectories of the traffic network are guaranteed to satisfy the desired LTL specification.

IV. Extensions to a Probabilistic Model

The above approach accommodates uncertainty in the disturbance input via nondeterminism in the finite state abstraction. However, in traffic networks, disturbance inputs are often characterized probabilistically. Furthermore, it is often sufficient for the controller to satisfy a specification with high probability. For example, we may wish to find a controller that avoids congestion with 95% probability. To this end, we extend the above methodology to a probabilistic framework using a probabilistic abstraction; computing such abstractions is an active area of research, *e.g.*, [16], [17].

We propose modifying the nondeterministic system developed in Section III to include transition probabilities obtained from a known probability distribution on the disturbance set $\mathcal{D}$. To this end, we obtain a Markov Decision Process (MDP) M from T' as follows:

Definition 3 (Traffic Model MDP)**.** *From an approximate transition system and a probability distribution on $\mathcal{D}$, we obtain a* Markov Decision Process (MDP) $M = (\mathbb{X}, \mathbb{S}, P)$ *that models the traffic dynamics where:*

- $\mathbb{X} = \prod_{\ell \in \mathcal{L}} \{1, \ldots, N_\ell\}$ *is the set of states,*
- $\mathbb{S} = \{0, 1\}^{|\mathcal{V}|}$ *is the set of controls,*
- $P : \mathbb{X} \times \mathbb{S} \times \mathbb{X} \to [0, 1]$ *is the transition probability function satisfying for all* $\mathbf{X} \in \mathbb{X}$ *and* $\mathbf{s} \in \mathbb{S}$, $\sum_{\mathbf{X}' \in \mathbb{X}} P(\mathbf{X}, \mathbf{s}, \mathbf{X}') = 1$ *and* $P(\mathbf{X}, \mathbf{s}, \mathbf{X}') > 0$ *if and only if* $(\mathbf{X}, \mathbf{s}, \mathbf{X}') \in \to'$. *A formula for P is given subsequently.*

We assume $\mathcal{D}$ is a hyperrectangle, that is, $\mathcal{D} = \prod_{\ell \in \mathcal{L}} [\underline{d}^\ell, \bar{d}^\ell]$ for some $\underline{d}^\ell \leq \bar{d}^\ell$ for all $\ell \in \mathcal{L}$. Let $p^{\mathcal{D}} : \mathcal{D} \to [0, 1]$ be the probability distribution on $\mathcal{D}$ so that $\int_{\mathcal{D}} p^{\mathcal{D}}(\delta) d\delta = 1$. We assume $p^{\mathcal{D}}$ is a product distribution, that is, $p^{\mathcal{D}}(d) = \prod_{\ell \in \mathcal{L}} p^{\mathcal{D}}_\ell(d_\ell)$ where $p^{\mathcal{D}}_\ell : [\underline{d}^\ell, \bar{d}^\ell] \to [0, 1]$ and $\int_{\underline{d}^\ell}^{\bar{d}^\ell} p^{\mathcal{D}}_\ell(\delta) d\delta = 1$ for all $\ell \in \mathcal{L}$. We suppose the exogenous disturbance is drawn independently from $\mathcal{D}$ at each time step. Define $p^{\mathbf{X},\mathbf{s}}_\ell : [0, x^{\text{cap}}_\ell] \to [0, 1]$ as follows:

$$\underline{y}_\ell = F_\ell(\underline{\mathbf{x}}^{\text{loc}}_\ell, \mathbf{s}^{\text{loc}}_\ell, 0), \quad \bar{y}_\ell = F_\ell(\overline{\mathbf{x}}^{\text{loc}}_\ell, \mathbf{s}^{\text{loc}}_\ell, 0), \quad (32)$$

$$p^{\mathbf{X},\mathbf{s}}_\ell(x'_\ell) = \tfrac{1}{\bar{y}_\ell - \underline{y}_\ell} \int_{\underline{y}_\ell}^{\bar{y}_\ell} p^{\mathcal{D}}_\ell(x'_\ell - z) dz \quad (33)$$

where $\underline{\mathbf{x}}^{\text{loc}}_\ell$, $\overline{\mathbf{x}}^{\text{loc}}_\ell$ are as given in (27)–(28). We substitute 0 for the disturbance in (32) as the disturbance is accommodated probabilistically in (33). Notice that $p^{\mathbf{X},\mathbf{s}}(x_\ell)$ has support only on $[\underline{y}_\ell + \underline{d}_\ell, \bar{y}_\ell + \bar{d}_\ell]$ and integrates to one on this domain. We interpret (33) as the probability distribution for the state of link ℓ in the next time step when initialized in discrete state $\mathbf{X}$. We define the joint probability distribution $p^{\mathbf{X},\mathbf{s}}(x) = \prod_{\ell \in \mathcal{L}} p^{\mathbf{X},\mathbf{s}}_\ell(x_\ell)$ and are now in a position to define the probability transition function P:

$$P(\mathbf{X}, \mathbf{s}, \mathbf{X}') := \int_{\mathbf{X}'} p^{\mathbf{X},\mathbf{s}}(x) dx. \quad (34)$$

We thus have a complete definition for an MDP model of the traffic network. For simple $p^{\mathcal{D}}(\cdot)$, P is straightforward to compute. Synthesizing a control policy for the MDP that maximizes the probability of satisfying an LTL specification has been explored in the literature, *e.g.*, [5, Ch. 10] and [11].

We emphasize that the results proposed in this section are preliminary. Unlike the nondeterministic case where a control strategy synthesized for the approximate transition system is guaranteed to apply to the original traffic network, the probability of satisfying a control objective computed from the MDP model may not completely reflect the actual satisfaction probability exhibited by the traffic network. This is due to the over-approximating nature of T' inherited by M and also due to the inherent Markov property of M. Nonetheless, these preliminary results are promising and suggest an important future role for a probabilistic framework in traffic control synthesis. For example, we may update estimated transition probabilities based on observed traffic flow, allowing us to incorporate measured data.

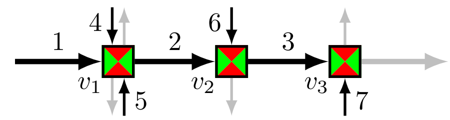


Fig. 1. Signalized network consisting of a major corridor road (links 1, 2, and 3) which intersects minor cross streets (links 4, 5, 6, and 7). The gray links are not explicitly modeled.

V. Example Network

We consider the example network in Fig. 1 which consists of a main corridor (links 1, 2, and 3) with intersecting cross streets (links 4, 5, 6, and 7) and three intersections. The gray links exit the network and are not explicitly modeled. Network parameters are given in Table I with time step 15 seconds. We divide the state of each link into partitions of length 10. We first assume that at each time step, $\mathbf{d}$ is such that one of the following four conditions holds: 0 to 20 vehicles join link 1, or 0 to 10 vehicles join links 4 and 5 each, or 0 to 10 vehicles join link 6, or 0 to 10 vehicles join link 7. If a disturbance input would result in a link exceeding its capacity, we assume the link state is set to capacity. This can be interpreted as excess vehicles choosing not to enter the network, but we could alternatively explicitly prevent this condition with appropriate choice of control specification.

We first consider the LTL property

$$\begin{aligned} \varphi_1 = &\Box\Diamond(s_{v_1} = 0) \wedge \Box\Diamond(s_{v_1} = 1) \wedge \Box\Diamond(s_{v_2} = 0) \wedge \quad (35)\\ &\Box\Diamond(s_{v_2} = 1) \wedge \Box\Diamond(s_{v_3} = 0) \wedge \Box\Diamond(s_{v_3} = 1) \wedge \quad (36)\\ &\Diamond\Box(x_2 \leq 30 \wedge x_3 \leq 30) \quad (37) \end{aligned}$$

with the following additional restriction for $v \in \{v_1, v_2, v_3\}$:

$$s_v[t] \neq s_v[t+1] \text{ implies } s_v[t+1] = s_v[t+2]. \quad (38)$$

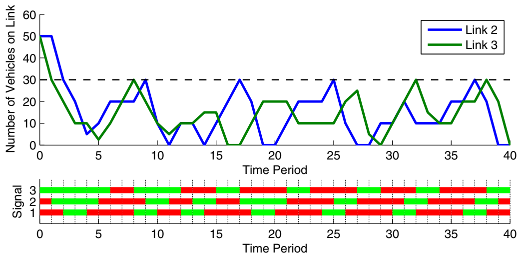
In words, (35)–(38) represent the following desired property: "(Always eventually each signal is red) and (always eventually each signal is green) and (eventually, links 2 and 3 have adequate supply for all future time) and (a signal's state cannot change twice in two periods)"

Above, "adequate supply" for links 2 and 3 means the number of vehicles on these links does not exceed 30, that is, these links can always accept upstream demand. The control objective (35)–(37) is transformed into a Rabin automaton with 18 states. To accommodate (35)–(38), we augment the discrete state space $\mathbb{X}$ with the signaling history from the previous two time steps; such a construction is standard and is required to accommodate control objectives that include the state of the signals themselves, which are modeled as inputs in the approximate quotient transition system T'. Fig. 2(a) shows a sample trajectory of the system with a control strategy synthesized using the approximate quotient system. The disturbance input is chosen as the maximum from one of the four sets described above to demonstrate the approach, and the particular set is chosen uniformly randomly. The control strategy is correct-by-construction and thus guaranteed to satisfy (35)–(38). The final automaton contained 76,800 discrete states and required approximately 14 minutes to obtain a solution.

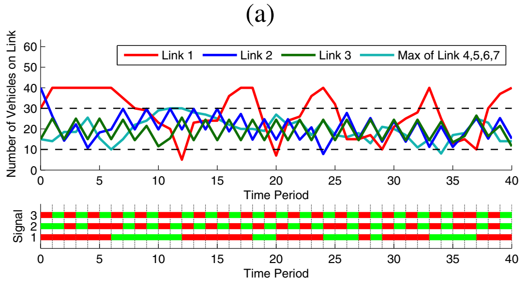
$$
\begin{aligned}
(x_1^{\text{cap}}, \dots, x_7^{\text{cap}}) &= (30, 50, 50, 20, 20, 20, 20)\\
(c_1, \dots, c_7) &= (10, 20, 20, 10, 10, 10, 10)\\
\beta_{12} = \beta_{23} = \beta_{42} = \beta_{52} = 0.5, &\qquad \beta_{63} = 1\\
\alpha_{42} = \alpha_{52} = 0.5, &\qquad \alpha_{12} = \alpha_{23} = \alpha_{63} = 1
\end{aligned}
$$

TABLE I

NETWORK PARAMETERS FOR THE EXAMPLE NETWORK IN SECTION V.



(a)



(b)

Fig. 2. (a) A sample trajectory resulting from the synthesized control policy that is guaranteed to satisfy the LTL policy φ_1 in (35)–(38). (b) A sample trajectory of the strategy obtained by modeling the traffic network as an MDP that satisfies φ_2 in (39)–(41). In the lower plots of both, green (red) for the signal trace indicates EW (NS) traffic is actuated.

We next consider a probabilistic abstraction and assume the disturbance is drawn from a uniform distribution over of the box $\mathcal{D} = \{\mathbf{d} \mid 0 \leq \mathbf{d} \leq \begin{bmatrix}20\ 0\ 0\ 10\ 10\ 10\ 10\end{bmatrix}^T\}$. We obtain a probabilistic abstraction of the dynamics in the form of an MDP as described in Section 3 with 432 discrete states. We wish to maximize the probability of satisfying the following temporal logic specification:

$$
\begin{aligned}
\varphi_2 =& \Diamond\Box(x_2 \leq 30 \wedge x_3 \leq 30) \wedge & (39)\\
& \Box\Diamond(x_4 \leq 10 \wedge x_5 \leq 10 \wedge x_6 \leq 10 \wedge x_7 \leq 10) \wedge & (40)\\
& \Box\big((x_1 > 30) \rightarrow \Diamond x_1 \leq 10\big). & (41)
\end{aligned}
$$

In words, (39)–(41) represent the following desired property: "(Eventually, links 2 and 3 have adequate supply for all future time) and (infinitely often, the queue on links 4, 5, 6, and 7 are short), and (whenever the queue on link 1 exceeds 30 vehicles, the queue eventually is short)."

We obtain a control strategy that achieves φ_2 with probability one as verified using the PRISM model-checker [18] (total computation time for synthesis was 23.0 seconds). On the other hand, if we model the system as a nondeterministic transition system as in Section III-C, we find that no controller exists satisfying φ_2, and thus the probabilistic formulation is crucial. We see in Fig. 2(b) that we indeed obtain a controller that satisfies φ_2 and behaves as expected.

VI. CONCLUSIONS

We have proposed a framework for synthesizing a control strategy for a network of signalized intersections that ensures the resulting traffic dynamics satisfy a control objective expressed as a linear temporal logic (LTL) formula. LTL is well-suited for modern transportation infrastructure where there are many, sometimes competing, objectives. Furthermore, we exploit structural properties of the traffic network to reduce the time required to compute a finite state abstraction of the dynamics. Our future research will address ways of further exploiting the structure inherent in such systems to reduce the number of discrete states in the abstraction.

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